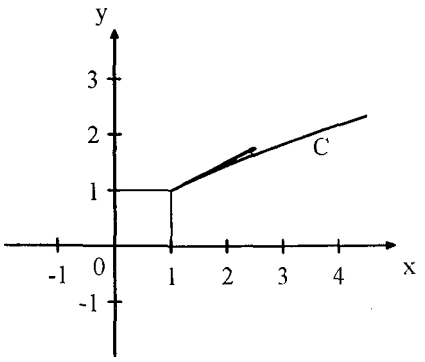
système d'équation $\begin{cases} y = f'_d(1)(x-1) + f(1) = \frac{1}{2}x + \frac{1}{2} \\ x \geq 1 \end{cases}$



II1)a) $\forall x \in]1;+\infty[$ on a $x^2 > x$.

$\forall t \in [x;x^2]$ on a :

$x \leq t \leq x^2 \Leftrightarrow \text{Log}x \leq \text{Log}t \leq \text{Log}(x^2)$

$\Leftrightarrow \frac{1}{\text{Log}(x^2)} \leq \frac{1}{\text{Log}t} \leq \frac{1}{\text{Log}x}$

D'où $\forall x \in]1;+\infty[; \int_x^{x^2} \frac{1}{\text{Log}(x^2)}dt \leq \int_x^{x^2} \frac{1}{\text{Log}t}dt \leq \int_x^{x^2} \frac{1}{\text{Log}x}dt$

$\Leftrightarrow \forall x \in]1;+\infty[; \frac{1}{\text{Log}(x^2)} \int_x^{x^2} 1dt \leq F(x) \leq \frac{1}{\text{Log}x} \int_x^{x^2} 1dt$

$\Leftrightarrow \forall x \in]1;+\infty[; \frac{x^2-x}{\text{Log}(x^2)} \leq F(x) \leq \frac{x^2-x}{\text{Log}x}$.

b) • $\forall x \in]1;+\infty[; \frac{x^2-x}{\text{Log}(x^2)} \leq F(x)$

de plus $\lim\limits_{x\to+\infty} \left[\frac{x^2-x}{\text{Log}(x^2)}\right] = \lim\limits_{x\to+\infty} \left[\frac{x(x-1)}{2\text{Log}x}\right]$

$= \lim\limits_{x\to+\infty} \frac{(x-1)}{2\frac{\text{Log}x}{x}} = +\infty$

D'où $\lim\limits_{x\to+\infty} F(x) = +\infty$.

• $\forall x \in]1;+\infty[; \frac{x^2-x}{\text{Log}(x^2)} \leq F(x) \leq \frac{x^2-x}{\text{Log}x}$

$\Leftrightarrow \forall x \in]1;+\infty[; \frac{x-1}{2\text{Log}x} \leq \frac{F(x)}{x} \leq \frac{x-1}{\text{Log}x}$

de plus $\lim\limits_{x\to+\infty} \frac{x-1}{2\text{Log}x} = \lim\limits_{x\to+\infty} \frac{1-\frac{1}{x}}{2\frac{\text{Log}x}{x}} = +\infty$

Donc $\lim\limits_{x\to+\infty} \frac{F(x)}{x} = +\infty$.